

Grade 8 Mathematics  
**Scope and Sequence**

**Quarter 1**

**Unit of Study 1.1: Transforming Geometric Figures Using Physical Models (10 days)**

***Standards for Mathematical Content***

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**Geometry**

**8.G**

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

- 8.G.1     Verify experimentally the properties of rotations, reflections, and translations:
- a.   Lines are taken to lines, and line segments to line segments of the same length.
  - b.   Angles are taken to angles of the same measure.
  - c.   Parallel lines are taken to parallel lines.

***Standards for Mathematical Practice***

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**4   Model with mathematics.**

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**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## Unit of Study 1.2: Proving Congruence and Similarity of Two-Dimensional Figures (6 days)

### *Standards for Mathematical Content*

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<b>Geometry</b>	<b>8.G</b>
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**Understand congruence and similarity using physical models, transparencies, or geometry software.**

- 8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- 8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
- 8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

### *Standards for Mathematical Practice*

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## Unit of Study 1.3: Informally Proving Angle Properties (5 days)

### *Standards for Mathematical Content*

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<b>Geometry</b>	<b>8.G</b>
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**Understand congruence and similarity using physical models, transparencies, or geometry software.**

- 8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, ~~and the angle-angle criterion for similarity of triangles~~. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

### *Standards for Mathematical Practice*

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#### **3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## Unit of Study 1.4: Understanding and Using Rational Numbers, Irrational Numbers, and Radicals (5 days)

### *Standards for Mathematical Content*

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<b>The Number System</b>	<b>8.NS</b>
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**Know that there are numbers that are not rational, and approximate them by rational numbers.**

- 8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
- 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g.,  $\pi^2$ ). *For example, by truncating the decimal expansion of  $\sqrt{2}$ , show that  $\sqrt{2}$  is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

<b>Expressions and Equations</b>	<b>8.EE</b>
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**Work with radicals and integer exponents.**

- 8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form  $x^2 = p$  and  $x^3 = p$ , where  $p$  is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that  $\sqrt{2}$  is irrational.

### *Standards for Mathematical Practice*

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#### **1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### **4 Model with mathematics.**

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#### **7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## Unit of Study 1.5: Explaining and Applying the Pythagorean Theorem and its Converse (15 days)

### *Standards for Mathematical Content*

<b>Geometry</b>	<b>8.G</b>
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#### **Understand and apply the Pythagorean Theorem.**

- 8.G.6 Explain a proof of the Pythagorean Theorem and its converse.
- 8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
- 8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

### *Standards for Mathematical Practice*

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**Quarter 2****Unit of Study 2.1: Solving Real-World Word Problems Involving Volume (5 days)*****Standards for Mathematical Content***

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**Geometry****8.G**

**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**

8.G.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

***Standards for Mathematical Practice***

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## Unit of Study 2.2: Knowing and Applying Radicals and Integer Exponents (15 days)

### *Standards for Mathematical Content*

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<b>Expressions and Equations</b>	<b>8.EE</b>
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#### **Work with radicals and integer exponents.**

- 8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example,  $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$ .
- 8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as  $3 \times 10^8$  and the population of the world as  $7 \times 10^9$ , and determine that the world population is more than 20 times larger.*
- 8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

### *Standards for Mathematical Practice*

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#### **2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### **3 Construct viable arguments and critique the reasoning of others.**

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### **8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 2.3: Graphing and Using Proportional Relationships, Lines, and Linear Equations (15 days)

### *Standards for Mathematical Content*

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Expressions and Equations	8.EE
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**Understand the connections between proportional relationships, lines, and linear equations.**

- 8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
- 8.EE.6 Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .

### *Standards for Mathematical Practice*

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## Quarter 3

### Unit of Study 3.1: Solving One-Variable Equations Transitioning into Two Variables (15 days)

#### *Standards for Mathematical Content*

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<b>Expressions and Equations</b>	<b>8.EE</b>
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**Analyze and solve linear equations and pairs of simultaneous linear equations.**

- 8.EE.7 Solve linear equations in one variable.
- a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form  $x = a$ ,  $a = a$ , or  $a = b$  results (where  $a$  and  $b$  are different numbers).
  - b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

<b>Statistics and Probability</b>	<b>8.SP</b>
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**Investigate patterns of association in bivariate data.**

- 8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

#### *Standards for Mathematical Practice*

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### **8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 3.2: Defining, Evaluating, and Comparing Functions (15 days)

### Standards for Mathematical Content

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<b>Functions</b>	<b>8.F</b>
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#### Define, evaluate, and compare functions.

- 8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.<sup>1</sup>
- <sup>1</sup> Function notation is not required in Grade 8.
- 8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*
- 8.F.3 Interpret the equation  $y = mx + b$  as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function  $A = s^2$  giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

### Standards for Mathematical Practice

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#### 1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### **3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

### **7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## Unit of Study 3.3: Using Functions to Model Relationships (15 days)

### *Standards for Mathematical Content*

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<b>Functions</b>	<b>8.F</b>
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#### **Use functions to model relationships between quantities.**

- 8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
- 8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

### *Standards for Mathematical Practice*

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#### **4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Quarter 4

### Unit of Study 4.1: Analyzing and Using Systems of Linear Equations with Word Problems (15 days)

#### *Standards for Mathematical Content*

#### Expressions and Equations

**8.EE**

#### Analyze and solve linear equations and pairs of simultaneous linear equations.

- 8.EE.8 Analyze and solve pairs of simultaneous linear equations.
- a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
  - b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example,  $3x + 2y = 5$  and  $3x + 2y = 6$  have no solution because  $3x + 2y$  cannot simultaneously be 5 and 6.*
  - c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

#### *Standards for Mathematical Practice*

## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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## Unit of Study 4.2: Using Scatterplots to Interpret Linear Associations (15 days)

### Standards for Mathematical Content

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<b>Statistics and Probability</b>	<b>8.SP</b>
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#### Investigate patterns of association in bivariate data.

- 8.SP.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
- 8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*
- 8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

### Standards for Mathematical Practice

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#### 4 Model with mathematics.

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## **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.



Algebra 1  
**Scope and Sequence**

**Quarter 1**

**Unit of Study 1.1: Looking at Number Sense  
(5 days)**

***Standards for Mathematical Content***

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**Quantities\***

**N-Q**

**Reason quantitatively and use units to solve problems.** [*Foundation for work with expressions, equations, and functions*]

- N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.\*
- N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.\*
- N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.\*

**The Real Number System**

**N-RN**

**Use properties of rational and irrational numbers.**

- N-RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

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## **Standards for Mathematical Practice**

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## Unit of Study 1.2: Interpreting Algebraic Expressions (7 days)

### Standards for Mathematical Content

#### Seeing Structure in Expressions

A-SSE

#### Interpret the structure of expressions [*Linear, exponential, quadratic*]

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.\*

- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ .*

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ . (Linear Only)*

#### Arithmetic with Polynomials and Rational Expressions

A-APR

#### Perform arithmetic operations on polynomials [*Linear and quadratic*]

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

### Standards for Mathematical Practice

#### 4 Model with mathematics.

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## **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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## Unit of Study 1.3: Operations with Linear Functions and Inequalities (15 days)

### Standards for Mathematical Content

Interpreting Functions	F-IF
<b>Understand the concept of a function and use function notation</b> [ <i>Learn as general principle; focus on linear and exponential and on arithmetic and geometric sequences</i> ]	
F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
F-IF.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. <i>For example, the Fibonacci sequence is defined recursively by <math>f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)</math> for <math>n \geq 1</math>.</i>
<b>Interpret functions that arise in applications in terms of the context</b> [ <i>Linear, exponential, and quadratic</i> ]	
F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*</i>
F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function <math>h(n)</math> gives the number of person-hours it takes to assemble <math>n</math> engines in a factory, then the positive integers would be an appropriate domain for the function.*</i>
F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
<b>Analyze functions using different representations</b> [ <i>Linear, exponential, quadratic, absolute value, step, piecewise-defined</i> ]	
F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.* <ol style="list-style-type: none"> <li>a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</li> </ol>

- F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

<b>Building Functions</b>	<b>F-BF</b>
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**Build a function that models a relationship between two quantities** [*For F.BF.1, 2, linear, exponential, and quadratic*]

- F-BF.1 Write a function that describes a relationship between two quantities.\*
- Determine an explicit expression, a recursive process, or steps for calculation from a context.
  - Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*

**Build new functions from existing functions** [*Linear, exponential, quadratic, and absolute value; for F.BF.4a, linear only*]

- F-BF.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

<b>Linear, Quadratic, and Exponential Models*</b>	<b>F-LE</b>
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**Construct and compare linear, quadratic, and exponential models and solve problems**

- F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.\*
- Prove that linear functions grow by equal differences over equal intervals, ~~and that exponential functions grow by equal factors over equal intervals.~~
  - Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).\*
- F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.\*

**Interpret expressions for functions in terms of the situation they model**

- F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.\*



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**Standards for Mathematical Practice**

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## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## Unit of Study 1.4: Solving Linear Equations and Inequalities (6 days)

### *Standards for Mathematical Content*

#### Reasoning with Equations and Inequalities

A-REI

#### Understand solving equations as a process of reasoning and explain the reasoning

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

#### Solve equations and inequalities in one variable

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

#### Represent and solve equations and inequalities graphically [*Linear and exponential; learn as general principle*]

A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

#### Creating Equations\*

A-CED

#### Create equations that describe numbers or relationships [*Equations using all available types of expressions, including simple root functions*]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear ~~and quadratic~~ functions, ~~and simple rational and exponential~~ functions.\**

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.\*

A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law  $V = IR$  to highlight resistance  $R$ .\**

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**Standards for Mathematical Practice**

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**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

**4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## Quarter 2

### Unit of Study 2.1: Solving Systems of Equations (15 days)

#### *Standards for Mathematical Content*

#### Reasoning with Equations and Inequalities

A-REI

##### Solve systems of equations [*Linear-linear and linear-quadratic*]

A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

##### Represent and solve equations and inequalities graphically [*Linear and exponential; learn as general principle*]

A-REI.11 Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.\*

A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

#### Creating Equations\*

A-CED

##### Create equations that describe numbers or relationships [*Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only*]

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*\*

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**Standards for Mathematical Practice**

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**3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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## Unit of Study 2.2: Introducing Quadratics (15 days)

### Standards for Mathematical Content

#### Seeing Structure in Expressions

A-SSE

#### Interpret the structure of expressions [~~Linear, exponential, quadratic~~]

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.\*

- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ .*

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .*

#### Write expressions in equivalent forms to solve problems [~~Quadratic and exponential~~]

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.\*

- Factor a quadratic expression to reveal the zeros of the function it defines.
- Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

### Standards for Mathematical Practice

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### **3 Construct viable arguments and critique the reasoning of others.**

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## Quarter 3

### Unit of Study 3.1: Continuing Quadratic/Polynomial Real-World Problems (15 days)

#### *Standards for Mathematical Content*

#### Arithmetic with Polynomials and Rational Expressions

A-APR

##### Perform arithmetic operations on polynomials [~~Linear and quadratic~~]

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A-APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

#### Creating Equations\*

A-CED

##### Create equations that describe numbers or relationships [~~Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only~~]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from ~~linear and quadratic functions, and simple rational and exponential functions.~~\**

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.\*

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*\*

#### Reasoning with Equations and Inequalities

A-REI

##### Solve equations and inequalities in one variable [~~Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions~~]

A-REI.4 Solve quadratic equations in one variable.

- a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

- b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

### **Represent and solve equations and inequalities graphically**

A-REI.11 Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are ~~linear,~~ polynomial, ~~rational,~~ absolute value, exponential, and logarithmic functions.\*

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## **Standards for Mathematical Practice**

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### **1 Make sense of problems and persevere in solving them.**

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## Unit of Study 3.2: Representing Polynomial Functions (10 days)

### Standards for Mathematical Content

#### Interpreting Functions

#### F-IF

**Understand the concept of a function and use function notation** [*Learn as general principle; focus on ~~linear and~~ exponential and on arithmetic and geometric sequences*]

- F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If  $f$  is a function and  $x$  is an element of its domain, then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .
- F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**Interpret functions that arise in applications in terms of the context** [*Linear, exponential, and quadratic*]

- F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*\*
- F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.*\*

**Analyze functions using different representations** [*Linear, exponential, quadratic, absolute value, step, piecewise-defined*]

- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*
- a. Graph ~~linear and~~ quadratic functions and show intercepts, maxima, and minima.
- F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

**Building Functions****F-BF**

**Build a function that models a relationship between two quantities** [*For F.BF.1, 2, linear, exponential, and quadratic*]

- F-BF.1 Write a function that describes a relationship between two quantities.\*
- Determine an explicit expression, a recursive process, or steps for calculation from a context.

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**Standards for Mathematical Practice**

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**4 Model with mathematics.**

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**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision

appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have.

## Unit of Study 3.3: Interpreting the Structure of Expressions and Using Equations and Inequalities to Solve Problems (10 days)

### Standards for Mathematical Content

#### Seeing Structure in Expressions

A-SSE

**Interpret the structure of expressions** [~~Linear~~, exponential, ~~quadratic~~]

A-SSE.1 Interpret expressions that represent a quantity in terms of its context.\*

- Interpret parts of an expression, such as terms, factors, and coefficients.
- Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ .*

A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .*

#### Creating Equations\*

A-CED

**Create equations that describe numbers or relationships** [~~Linear, quadratic, and exponential (integer inputs only)~~; for A.CED.3 linear only]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear ~~and quadratic functions~~, and simple rational ~~and exponential functions~~.*\*

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## Unit of Study 3.4: Interpreting and Analyzing Functions (10 days)

### Standards for Mathematical Content

#### Interpreting Functions

#### F-IF

**Interpret functions that arise in applications in terms of the context** [~~Linear~~, exponential, and quadratic]

- F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*\*
- F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.*\*
- F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.\*

**Analyze functions using different representations** [~~Linear~~, exponential, quadratic, absolute value, step, piecewise-defined]

- F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

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**Standards for Mathematical Practice**

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## Quarter 4

### Unit of Study 4.1: Comparing Linear, Quadratic, and Exponential Models (10 days)

#### Standards for Mathematical Content

#### Linear, Quadratic, and Exponential Models\*

F-LE

#### Construct and compare linear, quadratic, and exponential models and solve problems

- F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.\*
- a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression  $1.15^t$  can be rewritten as  $(1.15^{1/12})^{12t} \approx 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

#### Interpreting Functions

F-IF

#### Analyze functions using different representations [*Linear, exponential, quadratic, absolute value, step, piecewise-defined*]

- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*
- b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

#### Building Functions

F-BF

#### Build new functions from existing functions [*Linear and quadratic*]

- F-BF.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

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**Standards for Mathematical Practice**

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**4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Unit of Study 4.2: Representing Data and Statistics with One Variable (15 days)

### *Standards for Mathematical Content*

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<b>Interpreting Categorical and Quantitative Data*</b>	<b>S-ID</b>
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**Summarize, represent, and interpret data on a single count or measurement variable**

- S-ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).\*
- S-ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.\*
- S-ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).\*

**Summarize, represent, and interpret data on two categorical and quantitative variables**

*[Linear focus, discuss general principle]*

- S-ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.\*

### *Standards for Mathematical Practice*

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## **5 Use appropriate tools strategically.**

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## Unit of Study 4.3: Interpreting Linear Models with Data (15 days)

### *Standards for Mathematical Content*

<b>Interpreting Categorical and Quantitative Data*</b>	<b>S-ID</b>
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**Summarize, represent, and interpret data on two categorical and quantitative variables**  
[*Linear focus, discuss general principle*]

- S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.\*
- a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
  - b. Informally assess the fit of a function by plotting and analyzing residuals.
  - c. Fit a linear function for a scatter plot that suggests a linear association.

#### **Interpret linear models**

- S-ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.\*
- S-ID.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.\*
- S-ID.9 Distinguish between correlation and causation.\*

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## **7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .



# Geometry

## Scope and Sequence

### Quarter 1

#### Unit of Study 1.1: Introducing Geometric Basics (14 days)

##### *Standards for Mathematical Content*

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<b>Congruence</b>	<b>G-CO</b>
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##### **Experiment with transformations in the plane**

G-CO.1 Know precise definitions of angle, ~~circle~~, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, ~~and distance around a circular arc~~.

##### **Prove geometric theorems**

G-CO.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

##### **Make geometric constructions**

G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*

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**Standards for Mathematical Practice**

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## Unit of Study 1.2: Proving Geometric Theorems Algebraically (6 days)

### *Standards for Mathematical Content*

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#### Expressing Geometric Properties with Equations

#### G-GPE

**Use coordinates to prove simple geometric theorems algebraically** [*Include distance formula; relate to Pythagorean theorem*]

G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point  $(1, \sqrt{3})$  lies on the circle centered at the origin and containing the point  $(0, 2)$ .

### *Standards for Mathematical Practice*

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#### **1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### **3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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#### **8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 1.3: Proving Triangles are Congruent (19 days)

### *Standards for Mathematical Content*

<b>Congruence</b>	<b>G-CO</b>
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#### **Understand congruence in terms of rigid motions**

G-CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions

G-CO.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; ~~the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.~~

<b>Expressing Geometric Properties with Equations</b>	<b>G-GPE</b>
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#### **Use coordinates to prove simple geometric theorems algebraically**

G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

G-GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G-GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.\*

### *Standards for Mathematical Practice*

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## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments..

## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**Quarter 2****Unit of Study 2.1: Proving Triangles are Similar (15 days)*****Standards for Mathematical Content***

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**Similarity, Right Triangles, and Trigonometry****G-SRT****Understand similarity in terms of similarity transformations**

G-SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor:

- a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

G-SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

**Prove theorems involving similarity**

G-SRT.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

G-SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

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## ***Standards for Mathematical Practice***

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## Unit of Study 2.2: Solving Right Triangles Using Trigonometric Ratios (14 days)

### *Standards for Mathematical Content*

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<b>Similarity, Right Triangles, and Trigonometry</b>	<b>G-SRT</b>
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#### **Define trigonometric ratios and solve problems involving right triangles**

G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.

G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.\*

### *Standards for Mathematical Practice*

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## Unit of Study 2.3: Using Descriptions of Rigid Motion to Transform Geometric Figures (11 days)

### *Standards for Mathematical Content*

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<b>Congruence</b>	<b>G-CO</b>
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#### **Experiment with transformations in the plane**

- G-CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
- G-CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
- G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
- G-CO.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

#### **Understand congruence in terms of rigid motions**

- G-CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
- G-CO.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

#### **Prove geometric theorems**

- G-CO.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

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## **Standards for Mathematical Practice**

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### **2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### **4 Model with mathematics.**

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### **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Quarter 3

### Unit of Study 3.1: Explaining Volume Formulas and Modeling Geometric Shapes (11 days)

#### *Standards for Mathematical Content*

<b>Geometric Measurement and Dimension</b>	<b>G-GMD</b>
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**Explain volume formulas and use them to solve problems**

G-GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*

G-GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

G-GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.\*

**Visualize relationships between two-dimensional and three-dimensional objects**

G-GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

<b>Modeling with Geometry</b>	<b>G-MG</b>
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**Apply geometric concepts in modeling situations**

G-MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).\*

G-MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).\*

G-MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).\*

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## ***Standards for Mathematical Practice***

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### **1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### **3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

### **4 Model with mathematics.**

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## Unit of Study 3.2: Using Definitions and Constructing Figures Inscribed in a Circle (15 days)

### *Standards for Mathematical Content*

<b>Congruence</b>	<b>G-CO</b>
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#### **Experiment with transformations in the plane**

G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, ~~and distance around a circular arc.~~

#### **Make geometric constructions** [*Formalize and explain processes*]

G-CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

### *Standards for Mathematical Practice*

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## **8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 3.3: Using Theorems of Circles (12 days)

### *Standards for Mathematical Content*

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<b>Circles</b>	<b>G-C</b>
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#### **Understand and apply theorems about circles**

- G-C.1 Prove that all circles are similar.
- G-C.2 Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
- G-C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
- G-C.4 (+) Construct a tangent line from a point outside a given circle to the circle.

### *Standards for Mathematical Practice*

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## Quarter 4

### Unit of Study 4.1: Arc Length and Equations of Circles and Parabolas (15 days)

#### *Standards for Mathematical Content*

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<b>Congruence</b>	<b>G-CO</b>
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**Make geometric constructions**

G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

<b>Circles</b>	<b>G-C</b>
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**Find arc lengths and areas of sectors of circles**

G-C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

<b>Expressing Geometric Properties with Equations</b>	<b>G-GPE</b>
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**Translate between the geometric description and the equation for a conic section**

G-GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G-GPE.2 Derive the equation of a parabola given a focus and directrix.

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**Standards for Mathematical Practice**

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**Unit of Study 4.2: Investigating Equations of Circles, and Parabolas  
(13 days)**

[Need standards]

## Unit of Study 4.3: Exploring the Laws of Sines and Cosines (10 days)

### *Standards for Mathematical Content*

<b>Similarity, Right Triangles, and Trigonometry</b>	<b>G-SRT</b>
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#### **Apply trigonometry to general triangles**

G-SRT.9 (+) Derive the formula  $A = 1/2 ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

G-SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.

G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

### *Standards for Mathematical Practice*

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#### **7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .



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Algebra 2  
**Scope and Sequence**

**Quarter 1**

**Unit of Study 1.1: Understanding Basic Parts of Expressions and Units (5 days)**

*Standards for Mathematical Content*

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**Quantities\***

**N-Q**

**Reason quantitatively and use units to solve problems.**

- N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.\*
- N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.\*
- N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.\*

**Seeing Structure in Expressions**

**A-SSE**

**Interpret the structure of expressions**

- A-SSE.1 Interpret expressions that represent a quantity in terms of its context.\*
- a. Interpret parts of an expression, such as terms, factors, and coefficients.
  - b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ .*
- A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .*

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**Standards for Mathematical Practice**

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## Unit of Study 1.2: Using Complex Number Systems (5 days)

### Standards for Mathematical Content

<b>The Complex Number System</b>	<b>N-CN</b>
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#### Perform arithmetic operations with complex numbers.

- N-CN.1 Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.
- N-CN.2 Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

#### Use complex numbers in polynomial identities and equations. [*Polynomials with real coefficients*]

- N-CN.7 Solve quadratic equations with real coefficients that have complex solutions.
- \*N-CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
- \*Teacher note: Algebra 2 PAP objective (Optional for Algebra 2 Regular)*

### Standards for Mathematical Practice

#### 2 Reason abstractly and quantitatively.

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#### 7 Look for and make use of structure.

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## Unit of Study 1.3: Introducing the Characteristics of Polynomial Functions Using Graphing (10 days)

### Standards for Mathematical Content

#### Arithmetic with Polynomials and Rational Expressions

**A-APR**

##### Perform arithmetic operations on polynomials [*Beyond quadratic*]

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

##### Understand the relationship between zeros and factors of polynomials

A-APR.2 Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .

A-APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

#### Interpreting Functions

**F-IF**

##### Interpret functions that arise in applications in terms of the context [*Emphasize selection of appropriate models*]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.\**

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.\*

##### Analyze functions using different representations [*Focus on using key features to guide selection of appropriate type of model function*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*

- b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

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**Standards for Mathematical Practice**

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**4 Model with mathematics.**

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**5 Use appropriate tools strategically.**

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**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .



## Unit of Study 1.4: Transforming, Solving Using the Intersections of Linear Functions and Solving for Zeros Using Graphs (15 days)

### Standards for Mathematical Content

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<b>Reasoning with Equations and Inequalities</b>	<b>A-REI</b>
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**Understand solving equations as a process of reasoning and explain the reasoning** [*Master linear*

**Represent and solve equations and inequalities graphically** [*Linear and exponential; learn as general principle*]

A-REI.11 Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.\*

<b>Building Functions</b>	<b>F-BF</b>
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**Build new functions from existing functions** [*Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types*]

F-BF.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

<b>Interpreting Functions</b>	<b>F-IF</b>
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**Analyze functions using different representations** [*Focus on using key features to guide selection of appropriate type of model function*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*

- c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

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**Standards for Mathematical Practice**

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**5 Use appropriate tools strategically.**

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**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Quarter 2

### Unit of Study 2.1: Working with Polynomial Functions (10 days)

#### *Standards for Mathematical Content*

#### Arithmetic with Polynomials and Rational Expressions

**A-APR**

##### Use polynomial identities to solve problems

A-APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.*

A-APR.5 (+) Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle.<sup>1</sup>

#### Creating Equations\*

**A-CED**

**Create equations that describe numbers or relationships** [*Equations using all available types of expressions, including simple root functions*]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.\**

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.\*

#### *Standards for Mathematical Practice*

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<sup>1</sup> The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

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## **8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 2.2: Understanding Constraints with Models Involving Equations and Inequalities (5 days)

### *Standards for Mathematical Content*

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<b>Quantities*</b>	<b>N-Q</b>
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#### **Reason quantitatively and use units to solve problems.**

- N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.\*
- N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.\*
- N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.\*

### *Standards for Mathematical Practice*

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#### **6 Attend to precision.**

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## Unit of Study 2.3: Solving Quadratic Equations Using Various Methods (10 days)

### Standards for Mathematical Content

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<b>Reasoning with Equations and Inequalities</b>	<b>A-REI</b>
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**Solve equations and inequalities in one variable** [*Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions*]

A-REI.4 Solve quadratic equations in one variable.

- b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

### Standards for Mathematical Practice

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## Unit of Study 2.4: Understanding Parabolas and Deriving Their Equations (5 days)

### *Standards for Mathematical Content*

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<b>Expressing Geometric Properties with Equations</b>	<b>G-GPE</b>
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**Translate between the geometric description and the equation for a conic section**

G-GPE.2 Derive the equation of a parabola given a focus and directrix.

### *Standards for Mathematical Practice*

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## Unit of Study 2.5: Using Systems of Equations to Solve for Points of Intersections (10 days)

### Standards for Mathematical Content

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<b>Reasoning with Equations and Inequalities</b>	<b>A-REI</b>
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**Solve systems of equations** [*Linear-linear and linear-quadratic*]

A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .*

<b>Creating Equations*</b>	<b>A-CED</b>
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**Create equations that describe numbers or relationships** [*Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only*]

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.\**

### Standards for Mathematical Practice

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## Quarter 3

### Unit of Study 3.1: Determining Inverse Functions (5 days)

#### *Standards for Mathematical Content*

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<b>Building Functions</b>	<b>F-BF</b>
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**Build new functions from existing functions**

F-BF.4 Find inverse functions.

- a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. For example,  $f(x) = 2x^3$  or  $f(x) = (x+1)/(x-1)$  for  $x \neq 1$ .

F-BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

#### *Standards for Mathematical Practice*

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## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well-remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## Unit of Study 3.2: Connecting Arithmetic and Geometric Sequences and Series with Linear and Exponent Functions (10 days)

### *Standards for Mathematical Content*

#### Seeing Structure in Expressions

A-SSE

##### Write expressions in equivalent forms to solve problems

A-SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*\*

#### Interpreting Functions

F-IF

##### Understand the concept of a function and use function notation

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$  for  $n \geq 1$ .*

#### Building Functions

F-BF

##### Build a function that models a relationship between two quantities [For F.BF.1, 2, linear, exponential, and quadratic]

- F-BF.1 Write a function that describes a relationship between two quantities.\*
- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
- F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.\*

#### Linear, Quadratic, and Exponential Models\*

F-LE

##### Construct and compare linear, quadratic, and exponential models and solve problems

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).\*

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**Standards for Mathematical Practice**

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## Unit of Study 3.3: Connecting Exponential and Logarithmic Functions Using Multiple Representations (10 days)

### Standards for Mathematical Content

#### The Real Number System

N-RN

##### Extend the properties of exponents to rational exponents.

N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5^{(1/3)3}$  to hold, so  $(5^{1/3})^3$  must equal 5.*

N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

#### Seeing Structure in Expressions

A-SSE

##### Write expressions in equivalent forms to solve problems

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.\*

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression  $1.15^t$  can be rewritten as  $(1.15^{1/12})^{12t} \approx 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

#### Interpreting Functions

F-IF

##### Analyze functions using different representations

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ ,  $y = (1.2)^{t/10}$ , and classify them as representing exponential growth or decay.*

#### Building Functions

F-BF

##### Build a function that models a relationship between two quantities

F-BF.1 Write a function that describes a relationship between two quantities.\*

c. (+) Compose functions. *For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.*



- F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.\*

<b>Linear, Quadratic, and Exponential Models*</b>	<b>F-LE</b>
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**Construct and compare linear, quadratic, and exponential models and solve problems**

- F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.\*
- Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.\*
- F-LE.4 For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.\*

**Interpret expressions for functions in terms of the situation they model**

- F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.\*

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***Standards for Mathematical Practice***

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**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## **2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## **4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## Unit of Study 3.4: Using Arithmetic Operations and Graphing with Rational Expressions (15 days)

### *Standards for Mathematical Content*

<b>Arithmetic with Polynomials and Rational Expressions</b>	<b>A-APR</b>
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**Rewrite rational expressions** [*Linear and quadratic denominators*]

A-APR.6 Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

<b>Reasoning with Equations and Inequalities</b>	<b>A-REI</b>
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**Understand solving equations as a process of reasoning and explain the reasoning** [*Simple radical and rational*]

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

### *Standards for Mathematical Practice*

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## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well-remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## Quarter 4

### Unit of Study 4.1: Deriving Values of Special Angles on the Unit Circle and Graphing Trigonometric Functions (12 days)

#### Standards for Mathematical Content

<b>Trigonometric Functions</b>	<b>F-TF</b>
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**Extend the domain of trigonometric functions using the unit circle**

- F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

**Prove and apply trigonometric identities**

- F-TF.8 Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

<b>Interpreting Functions</b>	<b>F-IF</b>
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**Interpret functions that arise in applications in terms of the context**

- F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.\**

<b>Trigonometric Functions</b>	<b>F-TF</b>
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**Model periodic phenomena with trigonometric functions**

- F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.\*

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**Standards for Mathematical Practice**

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**1 Make sense of problems and persevere in solving them.**

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**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## **8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 4.2: Interpreting Data and Making Inferences Using Data (6 days)

### *Standards for Mathematical Content*

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<b>Interpreting Categorical and Quantitative Data*</b>	<b>S-ID</b>
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#### **Summarize, represent, and interpret data on a single count or measurement variable**

S-ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

#### **Summarize, represent, and interpret data on two categorical and quantitative variables** [Linear focus, discuss general principle]

- S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.\*
- a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
  - b. Informally assess the fit of a function by plotting and analyzing residuals.

<b>Making Inferences and Justifying Conclusions*</b>	<b>S-IC</b>
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#### **Understand and evaluate random processes underlying statistical experiments**

- S-IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.\*
- S-IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?\**

#### **Make inferences and justify conclusions from sample surveys, experiments, and observational studies**

- S-IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.\*
- S-IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.\*
- S-IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.\*
- S-IC.6 Evaluate reports based on data.\*



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**Standards for Mathematical Practice**

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**3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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## Unit 4.3: Applying Probability to Interpret Data (8 days)

### Standards for Mathematical Content

#### Conditional Probability and the Rules of Probability\*

S-CP

#### Understand independence and conditional probability and use them to interpret data

- S-CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).\*
- S-CP.2 Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent.\*
- S-CP.3 Understand the conditional probability of  $A$  given  $B$  as  $P(A \text{ and } B)/P(B)$ , and interpret independence of  $A$  and  $B$  as saying that the conditional probability of  $A$  given  $B$  is the same as the probability of  $A$ , and the conditional probability of  $B$  given  $A$  is the same as the probability of  $B$ .\*
- S-CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*\*
- S-CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*\*

### Standards for Mathematical Practice

#### 3 Construct viable arguments and critique the reasoning of others.

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#### **5 Use appropriate tools strategically.**

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## Unit 4.4: Using Rules of Probability to Compute Models (18 days)

### *Standards for Mathematical Content*

#### Conditional Probability and the Rules of Probability\*

S-CP

**Use the rules of probability to compute probabilities of compound events in a uniform probability model**

- S-CP.6 Find the conditional probability of  $A$  given  $B$  as the fraction of  $B$ 's outcomes that also belong to  $A$ , and interpret the answer in terms of the model.\*
- S-CP.7 Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.\*
- S-CP.8 (+) Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model.\*
- S-CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.\*

### *Standards for Mathematical Practice*

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#### **7 Look for and make use of structure.**

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## Unit 4.5: Using Probability to Evaluate Outcomes (6 days)

### Standards for Mathematical Content

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#### Using Probability to Make Decisions\*

S-MD

Use probability to evaluate outcomes of decisions [*Introductory; apply counting rules*]

S-MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).\*

S-MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).\*

### Standards for Mathematical Practice

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Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.



Pre-AP Algebra 2  
**Scope and Sequence**

**Quarter 1**

**Unit of Study 1.1: Understanding Basic Parts of Expressions and Units (5 days)**

***Standards for Mathematical Content***

**Quantities\***

**N-Q**

**Reason quantitatively and use units to solve problems.**

- N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.\*
- N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.\*
- N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.\*

**Seeing Structure in Expressions**

**A-SSE**

**Interpret the structure of expressions**

- A-SSE.1 Interpret expressions that represent a quantity in terms of its context.\*
- Interpret parts of an expression, such as terms, factors, and coefficients.
  - Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ .*
- A-SSE.2 Use the structure of an expression to identify ways to rewrite it. *For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .*

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**Standards for Mathematical Practice**

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**6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Unit of Study 1.2: Using the Complex Number Systems (5 days)

### *Standards for Mathematical Content*

<b>The Complex Number System</b>	<b>N-CN</b>
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**Perform arithmetic operations with complex numbers.**

N-CN.1 Know there is a complex number  $i$  such that  $i^2 = -1$ , and every complex number has the form  $a + bi$  with  $a$  and  $b$  real.

N-CN.2 Use the relation  $i^2 = -1$  and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

**Use complex numbers in polynomial identities and equations.** [*Polynomials with real coefficients*]

N-CN.7 Solve quadratic equations with real coefficients that have complex solutions.

\*N-CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

*\*Teacher note: Algebra 2 PAP objective (Optional for Algebra 2 Regular)*

### *Standards for Mathematical Practice*

## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 1.3: Introducing the Characteristics of Polynomial Functions Using Graphing (10 days)

### Standards for Mathematical Content

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#### Arithmetic with Polynomials and Rational Expressions

**A-APR****Perform arithmetic operations on polynomials** [*Beyond quadratic*]

A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**Understand the relationship between zeros and factors of polynomials**

A-APR.2 Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .

A-APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

#### Interpreting Functions

**F-IF****Interpret functions that arise in applications in terms of the context** [*Emphasize selection of appropriate models*]

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*\*

F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.\*

**Analyze functions using different representations** [*Focus on using key features to guide selection of appropriate type of model function*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*

- b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

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**Standards for Mathematical Practice**

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## Unit of Study 1.4: Transforming, Solving Using the Intersections of Linear Functions and Solving for Zeros Using Graphs (15 days)

### Standards for Mathematical Content

#### Reasoning with Equations and Inequalities

**A-REI**

**Understand solving equations as a process of reasoning and explain the reasoning** [*Master linear*

**Represent and solve equations and inequalities graphically** [*Linear and exponential; learn as general principle*]

A-REI.11 Explain why the  $x$ -coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.\*

#### Building Functions

**F-BF**

**Build new functions from existing functions** [*Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types*]

F-BF.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

#### Interpreting Functions

**F-IF**

**Analyze functions using different representations** [*Focus on using key features to guide selection of appropriate type of model function*]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*

- c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

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## ***Standards for Mathematical Practice***

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### **4 Model with mathematics.**

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### **6 Attend to precision.**

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## Quarter 2

### Unit of Study 2.1: Working with Polynomial Functions (10 days)

#### *Standards for Mathematical Content*

#### Arithmetic with Polynomials and Rational Expressions

**A-APR**

##### Use polynomial identities to solve problems

A-APR.4 Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity  $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$  can be used to generate Pythagorean triples.*

A-APR.5 (+) Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle.<sup>1</sup>

#### Creating Equations\*

**A-CED**

**Create equations that describe numbers or relationships** [*Equations using all available types of expressions, including simple root functions*]

A-CED.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.\**

A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.\*

#### *Standards for Mathematical Practice*

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<sup>1</sup> The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

## **5 Use appropriate tools strategically.**

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## **8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 2.2: Understanding Constraints with Models Involving Equations and Inequalities (5 days)

### *Standards for Mathematical Content*

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<b>Quantities*</b>	<b>N-Q</b>
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#### **Reason quantitatively and use units to solve problems.**

- N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.\*
- N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.\*
- N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.\*

### *Standards for Mathematical Practice*

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## Unit of Study 2.3: Solving Quadratic Equations Using Various Methods (10 days)

### Standards for Mathematical Content

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<b>Reasoning with Equations and Inequalities</b>	<b>A-REI</b>
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**Solve equations and inequalities in one variable** [*Linear inequalities; literal that are linear in the variables being solved for; quadratics with real solutions*]

A-REI.4 Solve quadratic equations in one variable.

- b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

### Standards for Mathematical Practice

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## Unit of Study 2.4: Using Geometric Properties To Make A Visual Description Of A Quadratic Function (5 days)

### *Standards for Mathematical Content*

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<b>Expressing Geometric Properties with Equations</b>	<b>G-GPE</b>
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**Translate between the geometric description and the equation for a conic section**

G-GPE.2 Derive the equation of a parabola given a focus and directrix.

### *Standards for Mathematical Practice*

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## Unit of Study 2.5: Using Systems of Equations to Solve for Points of Intersections (10 days)

### Standards for Mathematical Content

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<b>Reasoning with Equations and Inequalities</b>	<b>A-REI</b>
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**Solve systems of equations** [*Linear-linear and linear-quadratic*]

A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A-REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line  $y = -3x$  and the circle  $x^2 + y^2 = 3$ .*

<b>Creating Equations*</b>	<b>A-CED</b>
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**Create equations that describe numbers or relationships** [*Linear, quadratic, and exponential (integer inputs only); for A.CED.3 linear only*]

A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.\**

### Standards for Mathematical Practice

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#### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.



## **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Quarter 3

### Unit of Study 3.1: Determining Inverse Functions (5 days)

#### *Standards for Mathematical Content*

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**Building Functions****F-BF****Build new functions from existing functions**

F-BF.4 Find inverse functions.

- a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. For example,  $f(x) = 2x^3$  or  $f(x) = (x+1)/(x-1)$  for  $x \neq 1$ .

F-BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

#### *Standards for Mathematical Practice*

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## 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

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## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well-remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## Unit of Study 3.2: Connecting Arithmetic and Geometric Sequences and Series with Linear and Exponent Functions (10 days)

### *Standards for Mathematical Content*

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<b>Seeing Structure in Expressions</b>	<b>A-SSE</b>
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**Write expressions in equivalent forms to solve problems**

A-SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.\**

<b>Interpreting Functions</b>	<b>F-IF</b>
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**Understand the concept of a function and use function notation**

F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by  $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$  for  $n \geq 1$ .*

<b>Building Functions</b>	<b>F-BF</b>
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**Build a function that models a relationship between two quantities** [*For F.BF.1, 2, linear, exponential, and quadratic*]

F-BF.1 Write a function that describes a relationship between two quantities.\*

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.\*

<b>Linear, Quadratic, and Exponential Models*</b>	<b>F-LE</b>
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**Construct and compare linear, quadratic, and exponential models and solve problems**

F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).\*

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**Standards for Mathematical Practice**

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**4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

**5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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## Unit of Study 3.3: Connecting Exponential and Logarithmic Functions Using Multiple Representations (10 days)

### Standards for Mathematical Content

#### The Real Number System

N-RN

#### Extend the properties of exponents to rational exponents.

N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5^{(1/3)3}$  to hold, so  $(5^{1/3})^3$  must equal 5.*

N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

#### Seeing Structure in Expressions

A-SSE

#### Write expressions in equivalent forms to solve problems

A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.\*

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression  $1.15^t$  can be rewritten as  $(1.15^{1/12})^{12t} \approx 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

#### Interpreting Functions

F-IF

#### Analyze functions using different representations

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ ,  $y = (1.2)^{t/10}$ , and classify them as representing exponential growth or decay.*

#### Building Functions

F-BF

#### Build a function that models a relationship between two quantities

F-BF.1 Write a function that describes a relationship between two quantities.\*

c. (+) Compose functions. *For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.*

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.\*

<b>Linear, Quadratic, and Exponential Models*</b>	<b>F-LE</b>
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**Construct and compare linear, quadratic, and exponential models and solve problems**

F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.\*

- a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.\*

F-LE.4 For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.\*

**Interpret expressions for functions in terms of the situation they model**

F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.\*

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***Standards for Mathematical Practice***

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**1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## **2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## **4 Model with mathematics.**

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## Unit of Study 3.4: Using Arithmetic Operations and Graphing with Rational Expressions (15 days)

### *Standards for Mathematical Content*

<b>Arithmetic with Polynomials and Rational Expressions</b>	<b>A-APR</b>
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**Rewrite rational expressions** [*Linear and quadratic denominators*]

A-APR.6 Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.

A-APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

<b>Reasoning with Equations and Inequalities</b>	<b>A-REI</b>
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**Understand solving equations as a process of reasoning and explain the reasoning** [*Simple radical and rational*]

A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A-REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

### *Standards for Mathematical Practice*

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## 5 Use appropriate tools strategically.

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## 7 Look for and make use of structure.

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## Quarter 4

### Unit of Study 4.1: Deriving Values of Special Angles on the Unit Circle and Graphing Trigonometric Functions (12 days)

#### Standards for Mathematical Content

<b>Trigonometric Functions</b>	<b>F-TF</b>
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**Extend the domain of trigonometric functions using the unit circle**

- F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

**Prove and apply trigonometric identities**

- F-TF.8 Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.

<b>Interpreting Functions</b>	<b>F-IF</b>
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**Interpret functions that arise in applications in terms of the context**

- F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.\**

<b>Trigonometric Functions</b>	<b>F-TF</b>
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**Model periodic phenomena with trigonometric functions**

- F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.\*

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## ***Standards for Mathematical Practice***

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### **1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

### **4 Model with mathematics.**

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### **5 Use appropriate tools strategically.**

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## **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## **8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 4.2: Interpreting Data and Making Inferences Using Data (6 days)

### *Standards for Mathematical Content*

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<b>Interpreting Categorical and Quantitative Data*</b>	<b>S-ID</b>
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**Summarize, represent, and interpret data on a single count or measurement variable**

S-ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

**Summarize, represent, and interpret data on two categorical and quantitative variables**  
[*Linear focus, discuss general principle*]

- S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.\*
- a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
  - b. Informally assess the fit of a function by plotting and analyzing residuals.

<b>Making Inferences and Justifying Conclusions*</b>	<b>S-IC</b>
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**Understand and evaluate random processes underlying statistical experiments**

- S-IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.\*
- S-IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?\**

**Make inferences and justify conclusions from sample surveys, experiments, and observational studies**

- S-IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.\*
- S-IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.\*
- S-IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.\*
- S-IC.6 Evaluate reports based on data.\*

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**Standards for Mathematical Practice**

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**3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## 8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through  $(1, 2)$  with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.



## Unit 4.3: Applying Probability to Interpret Data (8 days)

### Standards for Mathematical Content

#### Conditional Probability and the Rules of Probability\*

S-CP

#### Understand independence and conditional probability and use them to interpret data

- S-CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).\*
- S-CP.2 Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent.\*
- S-CP.3 Understand the conditional probability of  $A$  given  $B$  as  $P(A \text{ and } B)/P(B)$ , and interpret independence of  $A$  and  $B$  as saying that the conditional probability of  $A$  given  $B$  is the same as the probability of  $A$ , and the conditional probability of  $B$  given  $A$  is the same as the probability of  $B$ .\*
- S-CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*\*
- S-CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*\*

### Standards for Mathematical Practice

#### 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

#### **4 Model with mathematics.**

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#### **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## Unit 4.4: Using Rules of Probability to Compute Models (18 days)

### *Standards for Mathematical Content*

#### Conditional Probability and the Rules of Probability\*

S-CP

**Use the rules of probability to compute probabilities of compound events in a uniform probability model**

- S-CP.6 Find the conditional probability of  $A$  given  $B$  as the fraction of  $B$ 's outcomes that also belong to  $A$ , and interpret the answer in terms of the model.\*
- S-CP.7 Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.\*
- S-CP.8 (+) Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$ , and interpret the answer in terms of the model.\*
- S-CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.\*

### *Standards for Mathematical Practice*

#### **4 Model with mathematics.**

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#### **7 Look for and make use of structure.**

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## Unit 4.5: Using Probability to Compute Models (6 days)

### *Standards for Mathematical Content*

#### Using Probability to Make Decisions\*

S-MD

Use probability to evaluate outcomes of decisions [*Introductory; apply counting rules*]

S-MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).\*

S-MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).\*

### *Standards for Mathematical Practice*

#### 1 Make sense of problems and persevere in solving them.

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Precalculus  
**Scope and Sequence**

**Quarter 1**

**Unit of Study 1.1: Parent Functions and Transformations  
(5 days)**

***Standards for Mathematical Content***

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**Building Functions**

**F-BF**

**Build new functions from existing functions** [*Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types*]

F-BF.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

***Standards for Mathematical Practice***

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## Unit of Study 1.2: Rational Expressions, Functions, and Graphs (15 days)

### *Standards for Mathematical Content*

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<b>Interpreting Functions</b>	<b>F-IF</b>
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**Analyze functions using different representations** [*Focus on using key features to guide selection of appropriate type of model function*]

- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*
- d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

<b>Arithmetic with Polynomials and Rational Expressions</b>	<b>A-APR</b>
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**Rewrite rational expressions** [*Linear and quadratic denominators*]

- A-APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

### *Standards for Mathematical Practice*

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#### **4 Model with mathematics.**

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## **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## Unit of Study 1.3: Logarithmic and Exponential Equations, Functions, and Graphs (15 days)

### *Standards for Mathematical Content*

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<b>Interpreting Functions</b>	<b>F-IF</b>
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**Analyze functions using different representations** [*Focus on using key features to guide selection of appropriate type of model function*]

- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*
- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

<b>Building Functions</b>	<b>F-BF</b>
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**Build new functions from existing functions**

- F-BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

### *Standards for Mathematical Practice*

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#### **1 Make sense of problems and persevere in solving them.**

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#### **4 Model with mathematics.**

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## Unit of Study 1.4: Building Composite Functions (5 days)

### Standards for Mathematical Content

#### Reasoning with Equations and Inequalities

A-REI

##### Solve equations and inequalities in one variable

A-REI.4 Solve quadratic equations in one variable.

- Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

##### Analyze functions using different representations

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

#### Building Functions

F-BF

**Build a function that models a relationship between two quantities** [*Include all types of functions studied*]

F-BF.1 Write a function that describes a relationship between two quantities.\*

- (+) Compose functions. *For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.*

### Standards for Mathematical Practice

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## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## Quarter 2

### New Unit of Study 2.1: Expanding Binomials Using the Binomial Theorem (10 days)

#### *Standards for Mathematical Content*

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<b>Arithmetic with Polynomials and Rational Expressions</b>	<b>A-APR</b>
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#### **Use polynomial identities to solve problems.**

A-APR.5 (+) Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle.<sup>1</sup>

<sup>1</sup> The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

#### *Standards for Mathematical Practice*

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#### **2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions

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## Unit of Study 2.2: Performing Operations with Complex Numbers (15 days)

### *Standards for Mathematical Content*

<b>The Complex Number System</b>	<b>N-CN</b>
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**Perform arithmetic operations with complex numbers.**

N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

**Represent complex numbers and their operations on the complex plane.**

N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,  $(-1 + \sqrt{3}i)^3 = 8$  because  $(-1 + \sqrt{3}i)$  has modulus 2 and argument  $120^\circ$ .*

N-CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

**Use complex numbers in polynomial identities and equations.**

N-CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$ .*

### *Standards for Mathematical Practice*

### **3 Construct viable arguments and critique the reasoning of others.**

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

## 7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## 8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Unit of Study 2.3: Building Sequences and Series (15 days)

### Standards for Mathematical Content

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<b>Building Functions</b>	<b>F-BF</b>
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**Build a function that models a relationship between two quantities** [*Include all types of functions studied*]

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.\*

### Standards for Mathematical Practice

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#### 1 Make sense of problems and persevere in solving them.

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#### 4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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## Quarter 3

### Unit of Study 3.1: The Unit Circle and Right Triangle Trigonometry (10 days)

#### *Standards for Mathematical Content*

#### **Trigonometric Functions**

**F-TF**

#### **Extend the domain of trigonometric functions using the unit circle**

- F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F-TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi-x$ ,  $\pi+x$ , and  $2\pi-x$  in terms of their values for  $x$ , where  $x$  is any real number.
- F-TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

#### *Standards for Mathematical Practice*

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#### **8 Look for and express regularity in repeated reasoning.**

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## Unit of Study 3.2: Graphing Trigonometric Functions (10 days)

### *Standards for Mathematical Content*

<b>Interpreting Functions</b>	<b>F-IF</b>
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**Analyze functions using different representations** [~~Logarithmic~~ and trigonometric functions]

F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*

- e. Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude.

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#### **2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### **4 Model with mathematics.**

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## Unit of Study 3.3: Inverse Trigonometric Functions (10 days)

### *Standards for Mathematical Content*

<b>Building Functions</b>	<b>F-BF</b>
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#### **Build new functions from existing functions**

F-BF.4 Find inverse functions.

- b. (+) Verify by composition that one function is the inverse of another.
- c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
- d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

<b>Trigonometric Functions</b>	<b>F-TF</b>
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#### **Model periodic phenomena with trigonometric functions**

F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.\*

F-TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.\*

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## Unit of Study 3.4: Proving and Applying Trigonometric Identities (10 days)

### *Standards for Mathematical Content*

<b>Trigonometric Functions</b>	<b>F-TF</b>
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#### **Prove and apply trigonometric identities**

- F-TF.8 Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.
- F-TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

### *Standards for Mathematical Practice*

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## Quarter 4

### Unit of Study 4.1: Conic Sections—Ellipses and Hyperbolas (10 days)

#### *Standards for Mathematical Content*

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<b>Expressing Geometric Properties with Equations</b>	<b>G-GPE</b>
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**Translate between the geometric description and the equation for a conic section**

G-GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

#### *Standards for Mathematical Practice*

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#### **7 Look for and make use of structure.**

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## Unit of Study 4.2: Cavalieri's Principle (10 days)

### *Standards for Mathematical Content*

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<b>Geometric Measurement and Dimension</b>	<b>G-GMD</b>
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**Explain volume formulas and use them to solve problems**

G-GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

### *Standards for Mathematical Practice*

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## Unit of Study 4.3: Solving Systems of Equations with Matrices and Inverse Matrices (10 days)

### *Standards for Mathematical Content*

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<b>Reasoning with Equations and Inequalities</b>	<b>A-REI</b>
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#### **Solve systems of equations**

A-REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.

A-REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater).

### *Standards for Mathematical Practice*

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## Unit of Study 4.4: Performing Operations with Matrices (10 days)

### *Standards for Mathematical Content*

<b>Vector and Matrix Quantities</b>	<b>N-VM</b>
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#### **Perform operations on matrices and use matrices in applications.**

N-VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.

N-VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N-VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N-VM.12 (+) Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

### *Standards for Mathematical Practice*

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## **2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

## **4 Model with mathematics.**

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# Pre-Advanced Placement Precalculus Scope and Sequence

## Quarter 1

### Unit of Study 1.1: Parent Functions and Transformations (5 days)

#### *Standards for Mathematical Content*

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#### **Building Functions**

#### **F-BF**

**Build new functions from existing functions** [*Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types*]

F-BF.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $k f(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

#### *Standards for Mathematical Practice*

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## **7 Look for and make use of structure.**

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## Unit of Study 1.2: Rational Expressions, Functions, and Graphs (15 days)

### *Standards for Mathematical Content*

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<b>Interpreting Functions</b>	<b>F-IF</b>
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**Analyze functions using different representations** [*Focus on using key features to guide selection of appropriate type of model function*]

- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*
- d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

<b>Arithmetic with Polynomials and Rational Expressions</b>	<b>A-APR</b>
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**Rewrite rational expressions** [*Linear and quadratic denominators*]

- A-APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

### *Standards for Mathematical Practice*

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#### **4 Model with mathematics.**

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## **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.



## Unit of Study 1.3: Logarithmic and Exponential Equations, Functions, and Graphs (15 days)

### *Standards for Mathematical Content*

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<b>Interpreting Functions</b>	<b>F-IF</b>
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**Analyze functions using different representations** [*Focus on using key features to guide selection of appropriate type of model function*]

- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.\*
- e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

<b>Building Functions</b>	<b>F-BF</b>
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**Build new functions from existing functions**

- F-BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

### *Standards for Mathematical Practice*

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#### **1 Make sense of problems and persevere in solving them.**

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## Unit of Study 1.4: Building Composite Functions (5 days)

### Standards for Mathematical Content

#### Reasoning with Equations and Inequalities

**A-REI**

##### Solve equations and inequalities in one variable

A-REI.4 Solve quadratic equations in one variable.

- a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

##### Analyze functions using different representations

F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

#### Building Functions

**F-BF**

**Build a function that models a relationship between two quantities** [*Include all types of functions studied*]

F-BF.1 Write a function that describes a relationship between two quantities.\*

- c. (+) Compose functions. *For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.*

### Standards for Mathematical Practice

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**Quarter 2****Unit of Study 2.1: Building Sequences and Series (10 days)*****Standards for Mathematical Content***

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**Building Functions****F-BF**

**Build a function that models a relationship between two quantities** [*Include all types of functions studied*]

F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.\*

***Standards for Mathematical Practice***

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## Unit of Study 2.2: The Unit Circle and Right Triangle Trigonometry (5 days)

### *Standards for Mathematical Content*

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<b>Trigonometric Functions</b>	<b>F-TF</b>
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#### **Extend the domain of trigonometric functions using the unit circle**

- F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F-TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for  $\pi/3$ ,  $\pi/4$  and  $\pi/6$ , and use the unit circle to express the values of sine, cosine, and tangent for  $\pi-x$ ,  $\pi+x$ , and  $2\pi-x$  in terms of their values for  $x$ , where  $x$  is any real number.
- F-TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

### *Standards for Mathematical Practice*

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#### **8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.



## Unit of Study 2.3: Graphing Trigonometric Functions (5 days)

### Standards for Mathematical Content

#### Interpreting Functions

**F-IF**

Analyze functions using different representations [~~Logarithmic~~ and trigonometric functions]

- F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
- e. Graph ~~exponential and logarithmic functions, showing intercepts and end behavior, and~~ trigonometric functions, showing period, midline, and amplitude.

### Standards for Mathematical Practice

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#### 2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

#### **4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

#### **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## Unit of Study 2.4: Inverse Trigonometric Functions (10 days)

### *Standards for Mathematical Content*

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<b>Building Functions</b>	<b>F-BF</b>
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#### **Build new functions from existing functions**

F-BF.4 Find inverse functions.

- b. (+) Verify by composition that one function is the inverse of another.
- c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
- d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

<b>Trigonometric Functions</b>	<b>F-TF</b>
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#### **Model periodic phenomena with trigonometric functions**

F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.\*

F-TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.\*

### *Standards for Mathematical Practice*

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#### **1 Make sense of problems and persevere in solving them.**

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## **2 Reason abstractly and quantitatively.**

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## Unit of Study 2.5: Proving and Applying Trigonometric Identities (10 days)

### *Standards for Mathematical Content*

<b>Trigonometric Functions</b>	<b>F-TF</b>
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#### **Prove and apply trigonometric identities**

- F-TF.8 Prove the Pythagorean identity  $\sin^2(\theta) + \cos^2(\theta) = 1$  and use it to find  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  given  $\sin(\theta)$ ,  $\cos(\theta)$ , or  $\tan(\theta)$  and the quadrant of the angle.
- F-TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

### *Standards for Mathematical Practice*

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### **3 Construct viable arguments and critique the reasoning of others.**

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## Quarter 3

### Unit of Study 3.1: Conic Sections—Ellipses and Hyperbolas (10 days)

#### *Standards for Mathematical Content*

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<b>Expressing Geometric Properties with Equations</b>	<b>G-GPE</b>
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**Translate between the geometric description and the equation for a conic section**

G-GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

#### *Standards for Mathematical Practice*

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#### **7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

**8 Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.



## Unit of Study 3.2: Cavalieri's Principle (5 days)

### Standards for Mathematical Content

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<b>Geometric Measurement and Dimension</b>	<b>G-GMD</b>
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#### Explain volume formulas and use them to solve problems

G-GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

### Standards for Mathematical Practice

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## Unit of Study 3.3: Expanding Binomials Using the Binomial Theorem (10 days)

### *Standards for Mathematical Content*

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<b>Arithmetic with Polynomials and Rational Expressions</b>	<b>A-APR</b>
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#### **Use polynomial identities to solve problems.**

A-APR.5 (+) Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle.<sup>1</sup>

<sup>1</sup> The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

### *Standards for Mathematical Practice*

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#### **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions

## 7 Look for and make use of structure.

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## Unit of Study 3.4: Performing Operations with Complex Numbers (15 days)

### Standards for Mathematical Content

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<b>The Complex Number System</b>	<b>N-CN</b>
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**Perform arithmetic operations with complex numbers.**

N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

**Represent complex numbers and their operations on the complex plane.**

N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,  $(-1 + \sqrt{3}i)^3 = 8$  because  $(-1 + \sqrt{3}i)$  has modulus 2 and argument  $120^\circ$ .*

N-CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

**Use complex numbers in polynomial identities and equations.**

N-CN.8 (+) Extend polynomial identities to the complex numbers. *For example, rewrite  $x^2 + 4$  as  $(x + 2i)(x - 2i)$ .*

### Standards for Mathematical Practice

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### 3 Construct viable arguments and critique the reasoning of others.

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## Quarter 4

### Unit of Study 4.1: Solving Systems of Equations and Performing Operations with Matrices and Inverse Matrices (15 Days)

#### *Standards for Mathematical Content*

<b>Reasoning with Equations and Inequalities</b>	<b>A-REI</b>
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**Solve systems of equations**

A-REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.

A-REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension  $3 \times 3$  or greater).

<b>Vector and Matrix Quantities</b>	<b>N-VM</b>
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**Perform operations on matrices and use matrices in applications.**

N-VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.

N-VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

N-VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

N-VM.12 (+) Work with  $2 \times 2$  matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

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## **Standards for Mathematical Practice**

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### **1 Make sense of problems and persevere in solving them.**

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## 5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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## Unit of Study 4.2: Using Matrices in Real-World Applications (10 days)

### *Standards for Mathematical Content*

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<b>Vector and Matrix Quantities</b>	<b>N-VM</b>
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#### **Perform operations on matrices and use matrices in applications.**

N-VM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

N-VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

### *Standards for Mathematical Practice*

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## Unit of Study 4.3: Operations with Vectors (15 days)

### *Standards for Mathematical Content*

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<b>Vector and Matrix Quantities</b>	<b>N-VM</b>
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**Perform operations on vectors.**

N-VM.4 (+) Add and subtract vectors.

- Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
- Understand vector subtraction  $\mathbf{v} - \mathbf{w}$  as  $\mathbf{v} + (-\mathbf{w})$ , where  $-\mathbf{w}$  is the additive inverse of  $\mathbf{w}$ , with the same magnitude as  $\mathbf{w}$  and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N-VM.5 (+) Multiply a vector by a scalar.

- Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as  $c(v_x, v_y) = (cv_x, cv_y)$ .
- Compute the magnitude of a scalar multiple  $c\mathbf{v}$  using  $\|c\mathbf{v}\| = |c|\mathbf{v}$ . Compute the direction of  $c\mathbf{v}$  knowing that when  $|c|\mathbf{v} \neq 0$ , the direction of  $c\mathbf{v}$  is either along  $\mathbf{v}$  (for  $c > 0$ ) or against  $\mathbf{v}$  (for  $c < 0$ ).

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**Standards for Mathematical Practice**

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**2 Reason abstractly and quantitatively.**

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**7 Look for and make use of structure.**

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## Unit of Study 4.4: Representing and Modeling with Vector Quantities (5 days)

### *Standards for Mathematical Content*

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<b>Vector and Matrix Quantities</b>	<b>N-VM</b>
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**Represent and model with vector quantities.**

- N-VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g.,  $\mathbf{v}$ ,  $|\mathbf{v}|$ ,  $\|\mathbf{v}\|$ ,  $v$ ).
- N-VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- N-VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.

**Perform operations on matrices and use matrices in applications.**

- N-VM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

### *Standards for Mathematical Practice*

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#### **4 Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## **5 Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## **6 Attend to precision.**

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.