

Kindergarten Mathematics

Scope and Sequence

Quarter 1

Unit of Study 1.1: Developing Understanding of Numbers 0–5 (10 days)

Standards for Mathematical Content

Counting and Cardinality	K.CC
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Know number names and the count sequence.

K.CC.1 Count ~~to 100~~ by ones ~~and by tens~~.

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

- a. When counting objects, say the number names in the standard order, ~~pairing each object with one and only one number name and each number name with one and only one object.~~

Standards for Mathematical Practice

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Unit of Study 1.2: Writing and Representing Numbers 0–5 (10 days)

Standards for Mathematical Content

Counting and Cardinality	K.CC
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Know number names and the count sequence.

K.CC.3 Write numbers ~~from 0 to 20~~. Represent a number of objects with a written numeral ~~0–20~~ (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.5 Count to answer “how many?” questions about ~~as many as 20~~ things arranged in a line, a rectangular array, or a circle, or ~~as many as 10~~ things in a scattered configuration; ~~given a number from 1–20, count out that many objects.~~

Standards for Mathematical Practice

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Unit of Study 1.3: Developing Understanding of Numbers 0–10 (10 days)

Standards for Mathematical Content

Counting and Cardinality	K.CC
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Count to tell the number of objects.

- K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.
- a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name ~~and each number name with one and only one object.~~
 - b. Understand that the last number name said tells the number of objects counted. ~~The number of objects is the same regardless of their arrangement or the order in which they were counted.~~
 - c. Understand that each successive number name refers to a quantity that is one larger.

Standards for Mathematical Practice

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Unit of Study 1.4: Building, Drawing, and Analyzing Shapes (10 days)

Standards for Mathematical Content

Geometry	K.G
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Analyze, compare, create, and compose shapes.

K.G.5 Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.

Standards for Mathematical Practice

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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Quarter 2

Unit of Study 2.1: Developing an Understanding of Numbers 0–20 (10 days)

Standards for Mathematical Content

Counting and Cardinality

K.CC

Know number names and the count sequence.

K.CC.1 Count ~~to 100~~ by ones ~~and by tens~~.

Count to tell the number of objects.

K.CC.4 Understand the relationship between numbers and quantities; connect counting to cardinality.

- a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
- b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

K.CC.5 Count to answer “how many?” questions about as many as 20 things arranged in a line, ~~a rectangular array, or a circle, or as many as 10 things in a scattered~~ configuration; given a number from 1–20, count out that many objects.

Standards for Mathematical Practice

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Unit of Study 2.2: Writing and Representing Numbers 0–10 (10 days)

Standards for Mathematical Content

Counting and Cardinality	K.CC
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Know number names and the count sequence.

K.CC.3 Write numbers ~~from 0 to 20~~. Represent a number of objects with a written numeral ~~0–20~~ (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.5 Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, ~~or a circle, or as many as 10 things in a scattered configuration~~; given a number from 1–20, count out that many objects.

Standards for Mathematical Practice

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Unit of Study 2.3: Counting Forward to 20 From Any Number (10 days)

Standards for Mathematical Content

Counting and Cardinality	K.CC
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Know number names and the count sequence.

K.CC.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).

Standards for Mathematical Practice

6 Attend to precision.

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7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

Unit of Study 2.4: Identifying, Describing, and Comparing Attributes of Objects and 2-D Shapes (10 days)

Standards for Mathematical Content

Measurement and Data	K.MD
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Describe and compare measurable attributes.

- K.MD.1 Describe measurable attributes of objects, such as length or weight. ~~Describe several measurable attributes of a single object.~~
- K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, ~~and describe the difference.~~ *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Classify objects and count the number of objects in each category.

- K.MD.3 Classify objects into given categories; count the numbers of objects in each category ~~and sort the categories by count.~~³

³ Limit category counts to be less than or equal to 10.

Geometry	K.G
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Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

- K.G.1 Describe objects in the environment using names of shapes, ~~and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.~~
- K.G.2 Correctly name shapes regardless of their ~~orientations or overall size.~~

Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

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Quarter 3

Unit of Study 3.1: Developing Understanding of Numbers 0–50 and Introducing Counting by 10s (8 days)

Standards for Mathematical Content

Counting and Cardinality

K.CC

Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

8 Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Unit of Study 3.2: Writing and Representing Numbers 0–20 (8 days)

Standards for Mathematical Content

Counting and Cardinality	K.CC
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Know number names and the count sequence.

K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Count to tell the number of objects.

K.CC.5 Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Standards for Mathematical Practice

5 Use appropriate tools strategically.

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Unit of Study 3.3: Understanding Addition to Solve Word Problems (10 days)

Standards for Mathematical Content

Operations and Algebraic Thinking

K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

- K.OA.1 Represent addition ~~and subtraction~~ with objects, fingers, mental images, drawings², sounds (e.g., claps), acting out situations, ~~verbal explanations, expressions, or equations.~~
- ² Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
- K.OA.2 Solve addition ~~and subtraction~~ word problems, and add ~~and subtract~~ within 10, e.g., by using objects or drawings to represent the problem.
- K.OA.3 Decompose numbers less than or equal to 10 into pairs ~~in more than one way~~, e.g., by using objects or drawings, ~~and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).~~
- K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, ~~and record the answer with a drawing or equation.~~
- K.OA.5 ~~Fluently~~ add ~~and subtract~~ within 5.

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Unit of Study 3.4: Understanding and Using Subtraction to Solve Word Problems (10 days)

Standards for Mathematical Content

Operations and Algebraic Thinking	K.OA
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Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings², sounds (e.g., claps), acting out situations, ~~verbal explanations, expressions, or equations.~~

² Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.

K.OA.5 ~~Fluently~~ add and subtract within 5.

Standards for Mathematical Practice

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Unit of Study 3.5: Describing and Comparing 2-D and 3-D Shapes (4 days)

Standards for Mathematical Content

Measurement and Data	K.MD
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Describe and compare measurable attributes.

- K.MD.1 Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
- K.MD.2 Directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Classify objects and count the number of objects in each category.

- K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.³

³ Limit category counts to be less than or equal to 10.

Geometry	K.G
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Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

- K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
- K.G.2 Correctly name shapes regardless of their orientations or overall size.

Standards for Mathematical Practice

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

Quarter 4

Unit of Study 4.1: Counting from 0–100 Using Ones and Tens (5 days)

Standards for Mathematical Content

Counting and Cardinality	K.CC
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Know number names and the count sequence.

K.CC.1 Count to 100 by ones and by tens.

Standards for Mathematical Practice

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

Unit of Study 4.2: Adding and Subtracting up to 10 (10 days)

Standards for Mathematical Content

Operations and Algebraic Thinking	K.OA
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Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

- K.OA.1 Represent addition and subtraction with objects, fingers, mental images, drawings², sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
- ² Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)
- K.OA.2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
- K.OA.3 Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).
- K.OA.4 For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
- K.OA.5 Fluently add and subtract within 5.

Standards for Mathematical Practice

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Unit of Study 4.3: Developing Foundations for Place Value, 11–19 (10 days)

Standards for Mathematical Content

Number and Operations in Base Ten	K.NBT
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Work with numbers 11–19 to gain foundations for place value.

- K.NBT.1 Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., $18 = 10 + 8$); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Standards for Mathematical Practice

2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Unit of Study 4.4: Comparing Objects and Numbers (<, >, or =) Up to 10 (10 days)

Standards for Mathematical Content

Counting and Cardinality	K.CC
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Compare numbers.

K.CC.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.¹

¹ Include groups with up to ten objects.

Counting and Cardinality	K.CC
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Compare numbers.

K.CC.7 Compare two numbers between 1 and 10 presented as written numerals.

Standards for Mathematical Practice

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Unit of Study 4.5: Identifying, Analyzing, and Composing 2-D and 3-D Shapes (5 days)

Standards for Mathematical Content

Geometry	K.G
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Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

K.G.3 Identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”).

Analyze, compare, create, and compose shapes.

K.G.4 Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length).

K.G.6 Compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?”

Standards for Mathematical Practice

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